

Kondo-resonance, Coulomb blockade, and Andreev transport through a quantum dot

Kicheon Kang*

Max-Planck-Institut für Physik Komplexer Systeme, Nöthnitzer Str. 38, D-01187 Dresden, Germany
(February 1, 2008)

We study resonant tunneling through an interacting quantum dot coupled to normal metallic and superconducting leads. We show that large Coulomb interaction gives rise to novel effects in Andreev transport. Adopting an exact relation for the Green's function, we find that at zero temperature, the linear response conductance is enhanced due to Kondo-Andreev resonance in the Kondo limit, while it is suppressed in the empty site limit. In the Coulomb blocked region, on the other hand, the conductance is reduced more than the corresponding conductance with normal leads because large charging energy suppresses Andreev reflection.

PACS numbers: 73.23.Hk, 73.23.-b, 74.80.Fp

Electronic transport of mesoscopic devices containing superconducting electrode has been an interesting subject in recent years.¹ Transmission of electrons through normal metal - superconductor (N-S) interfaces requires the conversion of normal current to supercurrent, which is called Andreev reflection.² With the recent advances of nanofabrication technique, quantum interference effects have been extensively studied in the mesoscopic N-S heterostructures (see e.g. references in¹). In a phase coherent N-S structures, the phase of quasi-particles as well as Cooper pairs is preserved and transport properties depend strongly on the nature of quasiparticle phase. Theoretically, Landauer-Büttiker type formula^{3,4} has been used extensively to describe quantum transport in many kind of N-S hybrid structures using non-interacting models. (For review, see e.g.^{1,5}.)

Resonant tunneling through an interacting quantum dot (QD) or Anderson impurity has been intensively investigated recently. It has been shown that large Coulomb interaction gives rise to anomalous properties in transport. An example is the Kondo-resonant transport. Kondo-resonant transport has been predicted theoretically⁶⁻⁹ and verified experimentally¹⁰⁻¹² by conductance measurements for artificially made Anderson impurities. On the other side, strong electron-electron interactions suppress conductance peaks where the systems are weakly coupled to the leads. It has been shown that the electron-electron interactions lead to conductance suppression due to the orthogonality catastrophe.¹³⁻¹⁵ Stafford *et al.*¹⁵ showed that the coherent transmission in artificial molecule structures is suppressed with increasing the system size by using the Hubbard-type model. Coulomb interaction has been found to play a crucial role in the nature of the transmission phase.¹⁶⁻¹⁸ It has been shown that Coulomb interactions give rise to anomalous effects in phase evolution through a quantum dot embedded in an arm of the Aharonov-Bohm interferometer, such as an inter-resonance phase drop. Non-equilibrium transport in an interacting quantum dot where both leads are superconductors has been studied recently by using the non-equilibrium Green's function method.^{19,20} Andreev reflection has been supposed to be negligible in the

weak tunneling limit because large charging energy leads to Coulomb blockade of Andreev transport. In the meanwhile, for a moderately coupled quantum dot, multiple Andreev reflections give rise to a novel subgap structure in the current-voltage curve due to resonant tunneling, which are quite different from those of S-S contacts.¹⁹ Resonant Andreev tunneling in strongly correlated quantum dot coupled to normal and superconducting leads has been investigated recently by Fazio and Raimondi.²¹ Using the non-equilibrium Green's function formalism and equation of motion technique they have shown that the Kondo-resonant transmission is enhanced in the limit of large Coulomb repulsion due to the existence of a superconducting electrode.

In this paper, we investigate coherent transport through an interacting quantum dot coupled to normal and superconducting electrodes based on the scattering matrix formulation. We consider a model as shown schematically in Fig.1, where normal scattering and Andreev reflection are decoupled. In the QD-N₂ boundary, only normal scattering is taken into account while Andreev reflection is considered in the N₂-S boundary. It is assumed that the N₂-S boundary is perfect and the normal scattering doesn't occur at this boundary. This model is applicable to microjunctions where the length scale of normal scattering and Andreev reflection is well separated.²² With this model, Landauer type formula for the linear response conductance has been derived by Beenakker²² in the framework of non-interacting electron model. In the presence of interactions, this formula cannot be used in general because of the presence of inelastic processes. However, in the linear response regime ($V = 0$) with zero temperature, there is no phase space for inelastic processes and the formula can be equally applied to the system containing interactions. The linear response conductance for the system under consideration can be written as²²:

$$G_{NS} = \frac{4e^2}{h} \sum_n \frac{T_n^2}{(2 - T_n)^2}, \quad (1)$$

where T_n is transmission probability of n th channel. This equation is valid in the absence of an applied magnetic

field. We consider single channel case where the transmission probability through the quantum dot is represented by T_{QD} :

$$G_{NS} = \frac{4e^2}{h} \frac{T_{QD}^2}{(2 - T_{QD})^2}. \quad (2)$$

The corresponding formula for the normal leads is the well known Landauer formula³

$$G_N = \frac{2e^2}{h} T_{QD}. \quad (3)$$

Let's consider an Anderson impurity for the quantum dot with doubly degenerate level energy ε_0 and on-site Coulomb repulsion strength $U \sim e^2/C$, C being capacitance of the dot. At zero temperature the transmission probability T_{QD} can be obtained as follows owing to the fact that there are no inelastic processes.^{23,6} Due to the absence of the inelastic scattering, the imaginary part of the self-energy for the Green's function at the Fermi energy ε_F is given by

$$\text{Im} \Sigma(\varepsilon_F) = -\Gamma/2, \quad (4)$$

where $\Gamma = \Gamma_L + \Gamma_R$ and Γ_L/\hbar and Γ_R/\hbar are tunneling rate through left and right leads, respectively. With this condition the average occupation on the dot can be written as

$$\langle n \rangle = \frac{2}{\pi} \text{Im} [\log G^r(\varepsilon_F)]. \quad (5)$$

At zero temperature, the transmission probability can be expressed in terms of the exact Green's function as

$$T_{QD} = \Gamma_L \Gamma_R |G^r(\varepsilon_F)|^2, \quad (6)$$

which leads to the final expression with the help of the Eq.(4)

$$T_{QD} = \frac{4\Gamma_L \Gamma_R}{\Gamma^2} \sin^2 \varphi \quad (7)$$

where $\varphi = \pi \langle n \rangle / 2$. ($\langle n \rangle$ is average occupation of the dot.)

Here $\langle n \rangle$ is calculated numerically by an equation of motion method,²⁴ which has been shown to be quite accurate for large U . We consider a symmetric coupling of the quantum dot to leads, that is $\Gamma_L = \Gamma_R$. From the calculated values of the average occupation, we display the conductances in the Fig.2 as a function of $\varepsilon_F - \varepsilon_0$. The parameters used for calculations are $U = 50\Gamma$ and $W = 200\Gamma$, with W being the bandwidth of the leads. Since the transmission probability reaches one for $\varepsilon_F - \varepsilon_0 \gg \Gamma$, the conductance of normal-superconductor hybrid system goes to $4e^2/h$, which is twice of normal conductance. This is a result of perfect transmission through the quantum dot in the Kondo limit. On the contrary, the conductances are suppressed in the empty site limit because of small transparency.

G_{NS} decays faster than G_N because transmission by Andreev reflection requires two particle tunneling through the quantum dot.

In real systems, nearly perfect Kondo-resonant transmission could not be realized though it is predicted by an exact relation at zero temperature. When the temperature is larger than the Kondo temperature, Kondo-resonant transmission does not occur and the transport is suppressed by Coulomb repulsion, rather than enhanced. In the case $k_B T \ll \Gamma$, thermal broadening can be neglected and the conductances can be obtained in the following way with an approximate Green's function. If Kondo-like correlation is neglected, the transparency can be obtained by an approximate retarded Green's function²⁵ which is similar to the Breit-Wigner type

$$G^r(\varepsilon) = \frac{1 - \langle n \rangle / 2}{\varepsilon - \varepsilon_0 + i\Gamma/2} + \frac{\langle n \rangle / 2}{\varepsilon - \varepsilon_0 - U + i\Gamma/2}. \quad (8)$$

Note that this equation coincides with the Breit-Wigner formula for $U = 0$. The self-consistent value of $\langle n \rangle$ is given by the relation

$$\langle n \rangle = -\frac{2}{\pi} \int_{-\infty}^{\varepsilon_F} \text{Im} G^r(\varepsilon) d\varepsilon, \quad (9)$$

which leads to the expression¹⁸

$$\langle n \rangle = \frac{1 + 2P_1}{1 + P_1 - P_2}, \quad (10)$$

where

$$P_1 = \frac{1}{\pi} \arctan \frac{2(\varepsilon_F - \varepsilon_0)}{\Gamma},$$

$$P_2 = \frac{1}{\pi} \arctan \frac{2(\varepsilon_F - \varepsilon_0 - U)}{\Gamma}.$$

Then we can get the conductance through the Eq.(6)

Fig.3 displays the conductances obtained by the Eq.(6) and Eq.(8). As one can see, G_{NS} is suppressed more than G_N even in the "resonance" point. This phenomenon arises because Coulomb interactions in the dot suppress coherent transmission through the quantum dot. This would become a very general feature in transport through interacting system as far as the coupling to the leads are not so strong. If we consider larger system like a coupled chain of quantum dots, transmission probability will be more reduced due to orthogonality catastrophe. So one could say that in general G_{NS} will be negligible compared to G_N in strongly interacting systems weakly coupled to leads. Normal conductance suppression with increasing the system size has been studied by Stafford et al.¹⁵ While the normal conductance is proportional to the transparency, G_{NS} is second order of transmission probability. So G_{NS} will decrease faster than G_N with increasing the system size. Even in the case of single quantum dot, we could see suppression of transmission due to the Coulomb interaction from our calculations.

For comparison, we plot the conductances of non-interacting case ($U = 0$) in the Fig.4. As well known from the Breit-Wigner formula, one can see that $G_{NS} = 2G_N$ in the resonance point because of perfect transmission. Comparing Fig.3 and Fig.4, one can conclude that the large charging energy suppress Andreev reflection even on resonance.

In conclusion, we have discussed resonant tunneling through a strongly interacting quantum dot coupled to normal metallic and superconducting leads. We have found that in strongly interacting quantum dots, resonant Andreev transport is qualitatively different from that of non-interacting system. Based on the scattering matrix formalism and adopting an exact relation for the Green's function, we have shown that at zero temperature the linear response conductance is enhanced due to Kondo-Andreev resonance in the Kondo limit, while it is suppressed in the empty site limit. In the Coulomb blockaded region, on the other hand, the conductance is suppressed more than the corresponding normal conductance even in the resonance point, because large charging energy suppresses Andreev reflection.

The author thanks S. Kettelman and M. Leadbeater for discussions and comments on this manuscript. This work has been supported by KOSEF and in part by the visitors program of the MPI-PKS.

* Electronic address : kckang@mpipks-dresden.mpg.de

¹ C. J. Lambert and R. Raimondi, J. Phys.: Condens. Matt. **10**, 901 (1998).

² A. F. Andreev, Zh. Eksp. Teor. Fiz **46**, 1823 (1964). [Sov. Phys. JETP **19**, 1228 (1964)].

- ³ R. Landauer, Philos. Mag. **21**, 863 (1970).
- ⁴ M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).
- ⁵ C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
- ⁶ T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).
- ⁷ S. Hershfeld, J. H. Davies and J. W. Wilkins, Phys. Rev. Lett. **67**, 3720 (1991); Phys. Rev. B **46** (1992).
- ⁸ Y. Meir, N. S. Wingreen and P. A. Lee, **70**, 2601 (1993); N. S. Wingreen and Y. Meir, Phys. Rev. B **49**, 11 040 (1994).
- ⁹ A. L. Yeyati, A. Martin-Rodeo and F. Flores, Phys. Rev. Lett. **71**, 2991 (1993).
- ¹⁰ D. C. Ralph and R. A. Buhrman, Phys. Rev. Lett. **72**, 3401 (1994).
- ¹¹ D. Goldhaber-Gordon, H. Shtrikman, D. Abush-Magder, U. Meirav and M. A. Kastner, Nature **391**, 156 (1998).
- ¹² S. M. Cronenwell, T. H. Oosterkamp, and L. P. Kouwenhoven, unpublished (cond-mat/9804211).
- ¹³ J. M. Kinaret *et al.*, Phys. Rev. B **46**, 4681 (1992).
- ¹⁴ K. A. Matveev, L. I. Glazman, and H. U. Baranger, Phys. Rev. B **53**, 1034 (1996); J. M. Golden and B. I. Halperin, *ibid.* **53**, 3893 (1996).
- ¹⁵ C. A. Stafford, R. Kotlyar, and S. Das Sarma, unpublished (cond-mat/9801201).
- ¹⁶ C. Bruder, R. Fazio and H. Schoeller, Phys. Rev. Lett. **76**, 114 (1996).
- ¹⁷ Y. Oreg and Y. Gefen, Phys. Rev. B **55**, 13 726 (1997).
- ¹⁸ K. Kang, S. Y. Cho and C.-M. Ryu, submitted to Europhys. Lett.
- ¹⁹ A. L. Yeyati, J. C. Cuevas, A. López-Dávalos and A. Martín-Rodero, Phys. Rev. B **55**, R 6137 (1997).
- ²⁰ K. Kang, Phys. Rev. B **57**, 11 891 (1998).
- ²¹ R. Fazio and R. Raimondi, Phys. Rev. Lett. **80**, 2913 (1998).
- ²² C. W. J. Beenakker, Phys. Rev. B **46**, R12 841 (1992).
- ²³ D. C. Langreth, Phys. Rev. **150**, 516 (1966).
- ²⁴ K. Kang and B. I. Min, Phys. Rev. B **52**, 10 689 (1995).
- ²⁵ See e.g., H. Haug and A.-P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors*, Springer Series in Solid State Sciences, Vol.123 (Springer, Berlin, Heidelberg 1996) p.170-178.

FIG. 1. Schematic diagram of the quantum dot(QD) coupled to normal(N) and superconducting(S) leads. In the N₂-S interface, only Andreev reflection is considered.

FIG. 2. Conductance G_{NS} and G_N obtained by the Eq.(7) and numerical calculation of φ for $U = 50\Gamma$ and $W = 200\Gamma$.

FIG. 3. Conductance G_{NS} and G_N obtained by the Eq.(8) with (6) for $U = 50\Gamma$.

FIG. 4. Conductance G_{NS} and G_N in the non-interacting ($U = 0$) limit.

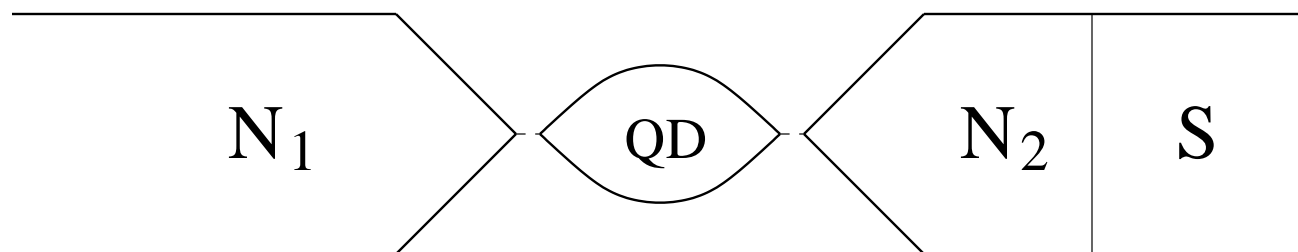


FIG. 1

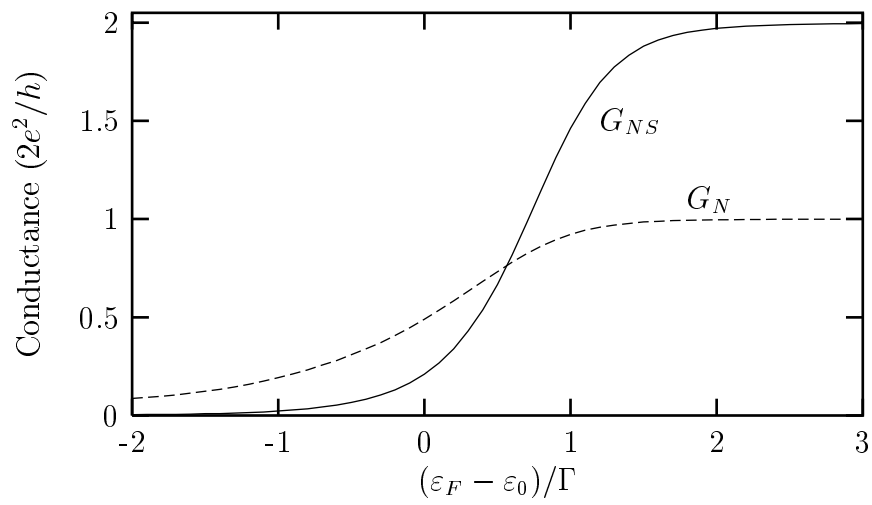


FIG. 2

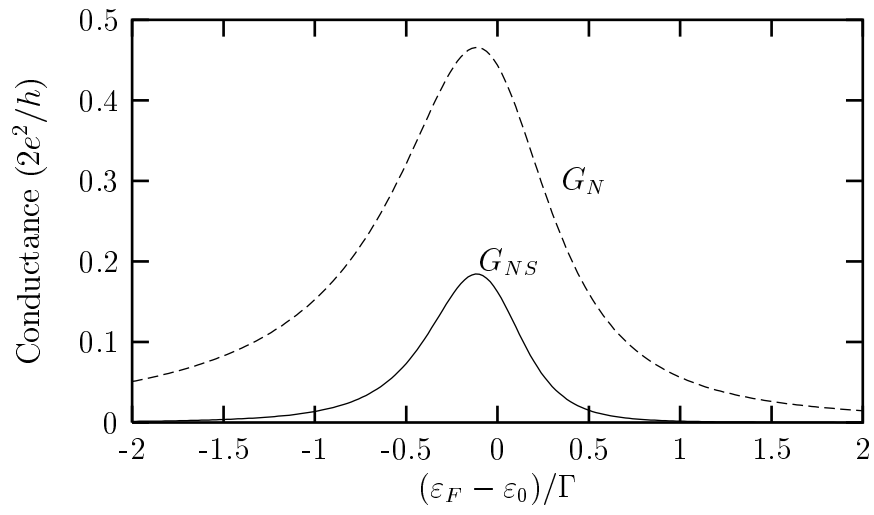


FIG. 3

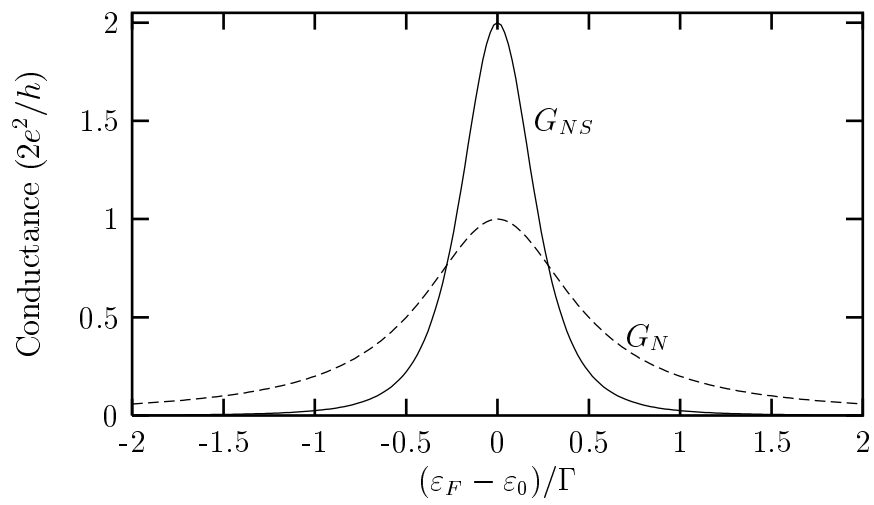


FIG. 4